1987 AB - 1

1a. $v = \int a = \int 6t - 18$

$= \frac{6t^2}{2} - 18t + c$

$= 3t^2 - 18t + c$

$v(0) = 24$

$24 = 3(0)^2 - 18(0) + c$

$c = 24$

$\therefore\ v = 3t^2 - 18t + 24$

b. at rest when $v = 0$

$0 = 3t^2 - 18t + 24$

$0 = t^2 - 6t + 8$

$0 = (t-4)(t-2)$

$t = 4 \quad t = 2$

c. $x = \int v = \int 3t^2 - 18t + 24$

$x = \frac{3t^3}{3} - \frac{18t^2}{2} + 24t + c$

$x = t^3 - 9t^2 + 24t + c$

$x(1) = 20$

$20 = 1^3 - 9(1)^2 + 24(1) + c$

$c = 4$

[illegible]

d. total distance

$t = 1$ to $t = 3$

turning point where $v = 0$

$\cancel{t = 4} \quad t = 2$

outside of domain

$t = 1 \quad x(1) = 1 - 9 + 24 + 4 = 20$

$t = 2 \quad x(2) = 8 - 36 + 48 + 4 = 24$

$t = 3 \quad x(3) = 27 - 81 + 72 + 4 = 22$

Total $= (1 \text{ to } 2) + (2 \text{ to } 3)$

$= (24 - 20) + (24 - 22)$

$= 4 + 2 = 6$

1987 AB - 2

2.a. Domain

$1-\sin x \geq 0$

$-\sin x \geq -1$

$\sin x \leq 1$

always true

$\therefore$ Domain $\mathbb{R}$

b. $f' = \frac{1}{2}(1-\sin x)^{-\frac{1}{2}}(-\cos x)$

c. $f' = \dfrac{-\cos x}{2\sqrt{1-\sin x}}$

$1-\sin x > 0$

$-\sin x > -1$

$\sin x < 1$

only problem pts. where

$\sin x = 1$

occurs at $x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$

$\therefore$ Domain $\mathbb{R}$ except $\frac{\pi}{2} + 2\pi n$ where n is an integer

d. $m_{tan} = y'$

$f^{-1}(0) = \dfrac{-\cos 0}{2\sqrt{1-\sin 0}} = \dfrac{-1}{2\sqrt{1}} = -\frac{1}{2}$

Find y at x

$y = \sqrt{1-\sin 0}$

$(0, 1)$

Eqn of line

$-\frac{1}{2} = \dfrac{y-1}{x-0}$

$-\frac{1}{2}x = y-1$

1987 #3

a. Find intersection point

$(64x)^{\frac{1}{4}} = x$

$64x = x^4$

$0 = x^4 - 64x$

$0 = x(x^3 - 64)$

$x = 0 \qquad x^3 - 64 = 0$

$x^3 = 64$

$x = 4$

$$\pi\int_0^4 \left((64x)^{\frac{1}{4}}\right)^2 - x^2$$

$$= \pi\int_0^4 (64x)^{\frac{1}{2}} - x^2$$

$$= \pi\int_0^4 8x^{\frac{1}{2}} - x^2$$

$$= \pi\left(\frac{8x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{3}\right]_0^4\Bigg)$$

$$= \pi\left(\frac{16}{3}x^{\frac{3}{2}} - \frac{x^3}{3}\right]_0^4$$

$$= \pi\left(\frac{16}{3}4^{\frac{3}{2}} - \frac{4^3}{3} - (0)\right)$$

$$= \pi\left(\frac{16}{3}\cdot 8 - \frac{64}{3}\right)$$

$$= \pi\left(\frac{128-64}{3}\right) = \frac{64\pi}{3}$$

b. $2\pi\int_0^4 x\left[(64x)^{\frac{1}{4}} - x\right]dx$

#4

a. $f' = 2\,\frac{1}{x^2+3}\cdot 2x - 1$

$$= \frac{4x}{x^2+3} - 1$$

$$= \frac{4x}{x^2+3} - \frac{x^2+3}{x^2+3}$$

$$= \frac{4x - x^2 - 3}{x^2+3}$$

$$= \frac{-x^2+4x-3}{x^2+3} = \frac{-(x^2-4x+3)}{x^2+3}$$

$$0 = \frac{-(x-3)(x-1)}{x^2+3}$$

		1		3	
$x-3$	—		—	0	
$x-1$	—	0			
x^2+3	+		+		+
f'	—	0	+	0	—
		MIN		MAX	

MINIMUM at $x=1$

MAXIMUM AT $x=3$

1987 #4

b. $$f'' = \frac{(x^2+3)(-2x+4)-(-x^2+4x-3)2x}{(x^2+3)^2}$$

$$= \frac{-2x^3+4x^2-6x+12-(-2x^3+8x^2-6x)}{(x^2+3)^2}$$

$$= \frac{\cancel{-2x^3}+\cancel{4x^2}\cancel{-6x}+12\cancel{+2x^3}-\cancel{8x^2}\cancel{+6x}}{(x^2+3)^2}$$

$$= \frac{-4x^2+12}{(x^2+3)^2}$$

$$0 = -4x^2+12$$
$$= -4(x^2-3)$$
$$= -4(x+\sqrt{3})(x-\sqrt{3})$$

		$-\sqrt{3}$		$\sqrt{3}$	
$x+\sqrt{3}$	−	0			
$x-\sqrt{3}$	−	−	−	0	
-4	−	−	−	−	−
f''	−	0	+	0	−
		sign change		sign change	

pts of inflection $x = \pm\sqrt{3}$

c. Absolulete maximum

test endpts and relative maximin

$f(-3) = 2\ln 12+3 \approx \ln 2892$ MAX at $2\ln 12+3 = 7.9698$

$f(5) = 2\ln 28-5 \approx \ln 5 = 1.66$

$f(3) = 2\ln 12-3 \approx \ln 7 = 1.96$

1987 #5

b. V when $\frac{1}{4}$ full $= \frac{15}{4}$

$$\frac{b}{h} = \frac{2}{3}$$

$$b = \frac{2}{3}h$$

$$V = \frac{1}{2} b h (5)$$

$$V = \frac{1}{2}\left(\frac{2}{3}h\right) h\, 5$$

$$V = \frac{5}{3}h^2$$

$$\frac{dV}{dt} = \frac{10}{3} h \frac{dh}{dt}$$

When $V = \frac{15}{4}$ find h

$$\frac{15}{4} = \frac{5}{3}h^2$$

$$\frac{9}{4} = h^2$$

$$h = \frac{3}{2}$$

$$\therefore \frac{dV}{dt} = \frac{10}{3}\left(\frac{3}{2}\right)\frac{dh}{dt}$$

$$-2 = 5\frac{dh}{dt}$$

$$\frac{-2}{5} = \frac{dh}{dt}$$

$$SA = \frac{1}{3}\,''$$

$$\frac{dSA}{dt} = \frac{10}{3}\frac{dh}{dt}$$

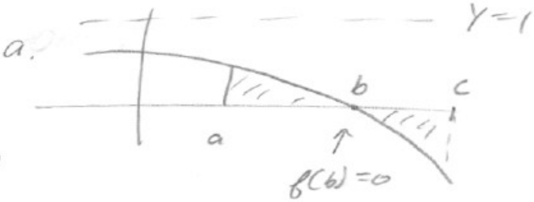
$$= \frac{10}{3}\left(\frac{-2}{5}\right)$$

$$= \frac{-4}{3} \text{ ft}^2/\text{min}$$

#6 a.

$f(x) < 1$

$f'(x) < 0$



$$\text{Area} = \int_a^b f(x) - \int_b^c f(x)\,dx$$

or $\int_a^b f(x) + \left|\int_c^b f(x)\right|$

b. $g' = \frac{(f(x)-1)0 - 1f'(x)}{(f(x)-1)^2} = \frac{-f'(x)}{[f(x)-1]^2}$

numerator will always be pos since $f'(x) < 0$

$\therefore$ $g(x)$ MUST BE INCREASING

c. $F(x) = h(f(x))$

TO TELL IF INC OR DEC FIND $F'(x)$

$$F'(x) = \underbrace{h'(f(x))}_{<0}\,\underbrace{f'(x)}_{<0} = \text{pos}$$

$\therefore$ ALWAYS INCREASING